

Heat and Mass transfer

Internal Flows

Internal flows and heat transfer (preliminaries)

Internal Flows

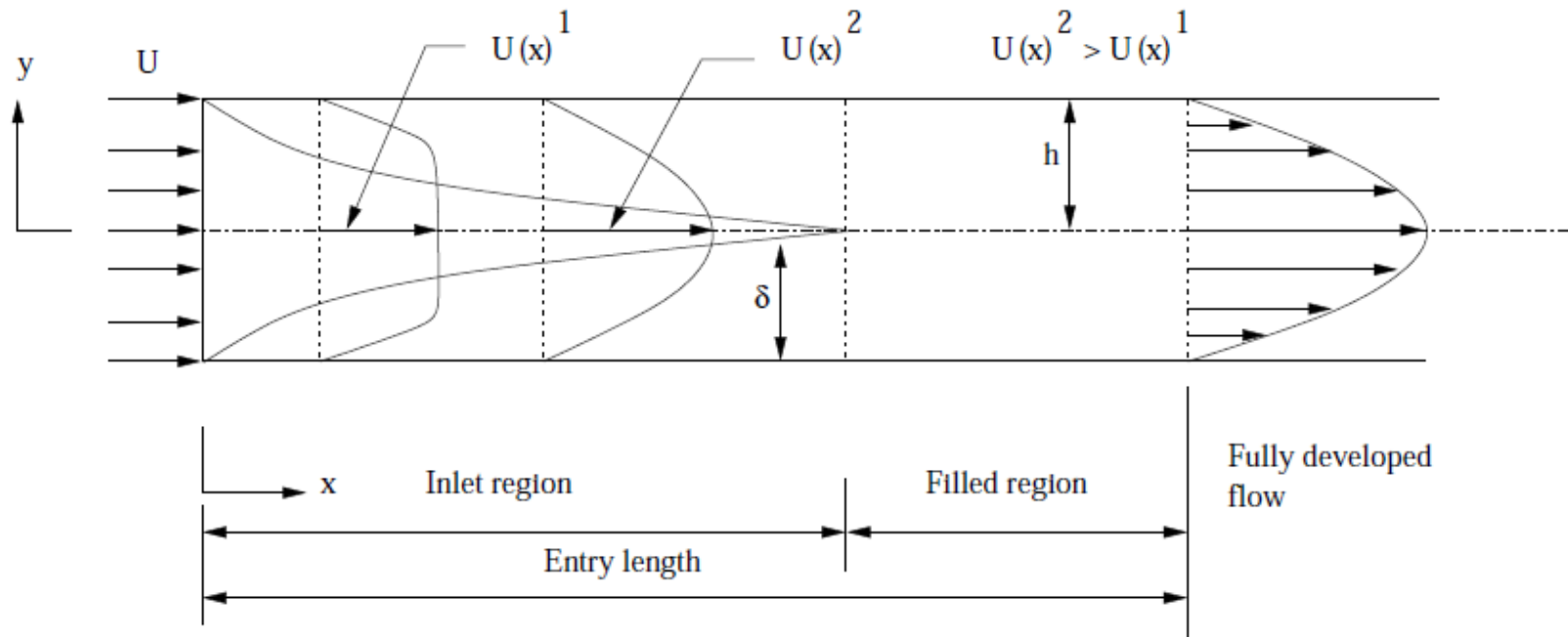
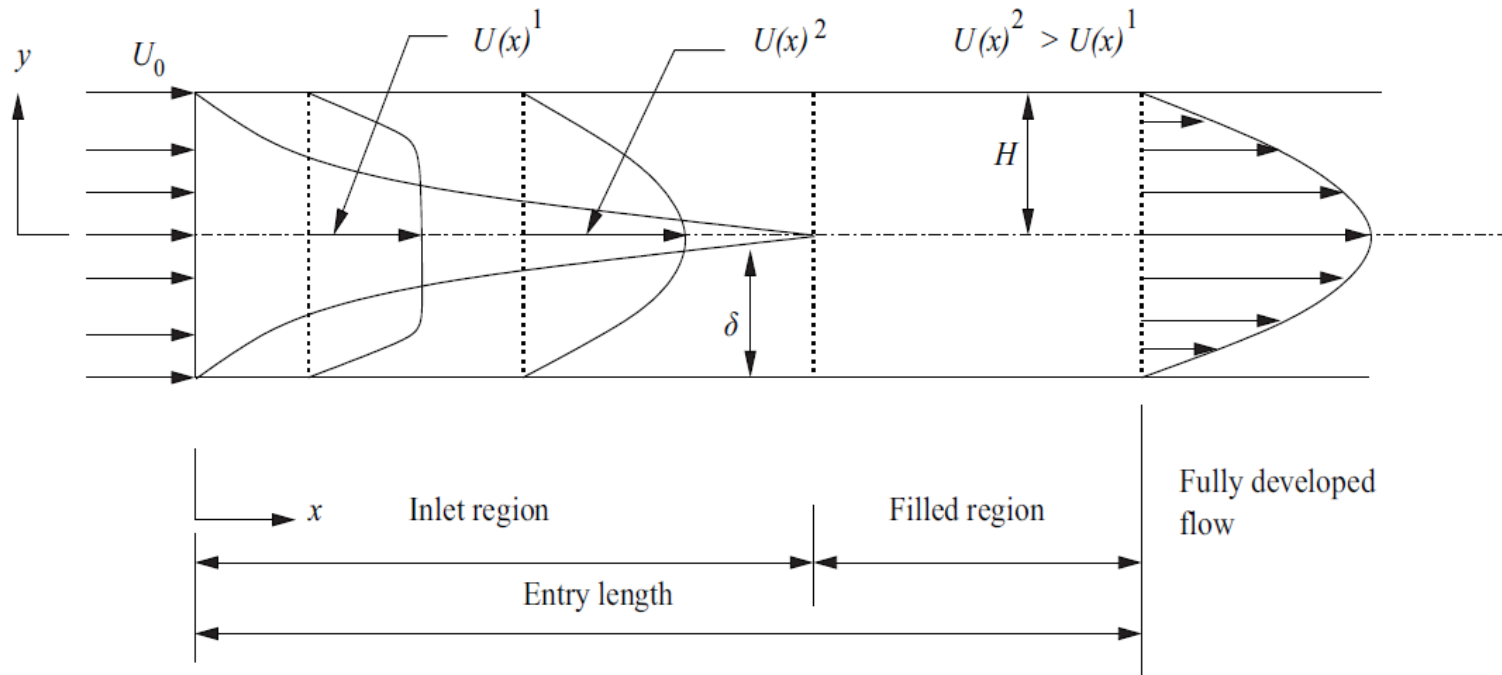


Figure 3.1: Flow development in a parallel plate channel

Introduction

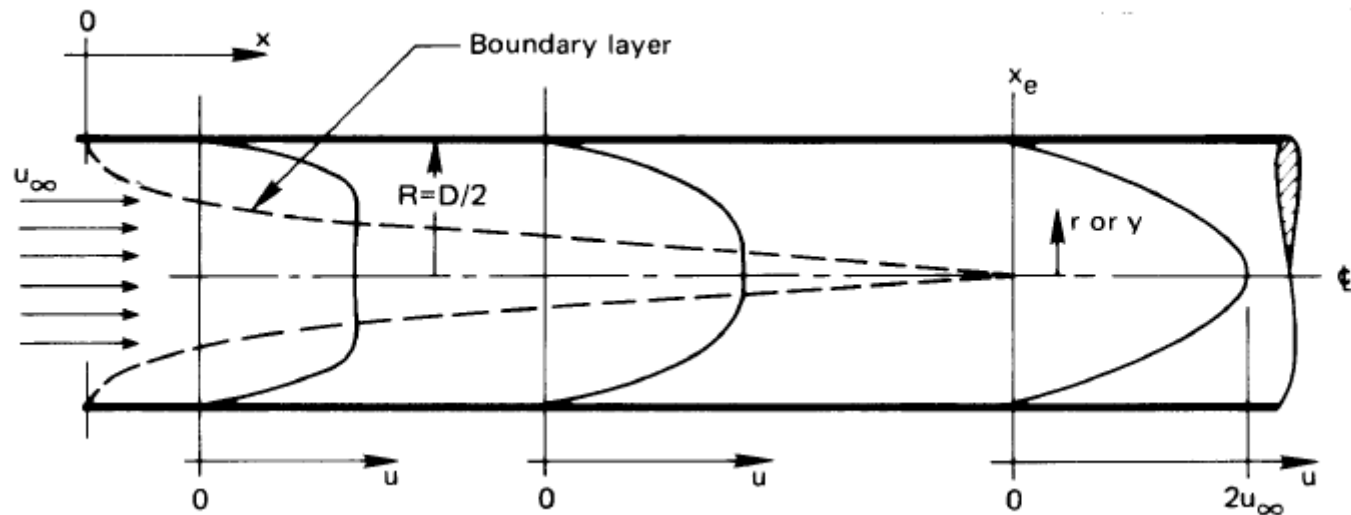
The flow development in a parallel plate channel has been shown in Figure 3.1. Beyond the entrance region, effect of viscosity extends over the entire cross section and the flow is said to be fully developed.



Owing to the growth of boundary-layer, the inviscid core accelerates. When the boundary-layers merge into each other, the acceleration is not stopped suddenly; rather it continues to persist and is destroyed gradually in the downstream till the fully developed velocity profiles appear ($\partial u / \partial x$ becomes zero) at this stage. The intermediate zone after the merger of the boundary-layers, through which the acceleration of the flow is gradually destroyed is known as the *filled* region. An invariant parabolic velocity profile is obtained beyond this filled zone.

Inlet zone and the filled zone together constitute the entrance region. The pressure gradient in the entrance region takes care of inertia effects and the wall shear and it varies with x . In the fully developed flow, the pressure gradient balances the wall shear stress only and has a constant value. The entry length, which constitutes both the inlet zone and the filled zone, can be estimated through a proper calculation of the filled zone and then by adding it to the inlet zone. Analysis of the filled zone is beyond the scope of this text.

Now we can take recourse to pipe flows. Like the earlier example of flow through a duct, the entrance region exists in the cases of pipe flows, too.



$$Re_D = \frac{\rho u_m D}{\mu} = \frac{\rho D^2 u_m^2}{\mu D u_m} = \frac{\rho D^2 u_m (D/t)}{D^2 \times \mu \frac{u_m}{D}} = \frac{\rho D^3 (u_m/t)}{D^2 \times \mu \frac{u_m}{D}} = \frac{\text{Inertia Force}}{\text{Viscous Force}}$$

U_m is the mean velocity of the fluid. The critical Reynolds number is usually considered as $Re_{D,C} = 2300$

Flow is turbulent for $Re_D > 4000$ (generally) and laminar for $Re_D \leq 2300$

In between is the transition zone. The estimation of the entrance length(x_e, h) can be done in the following way

$$\left(\frac{x_{e,h}}{D}\right)_{lam} \approx 0.05 Re_D$$

$$10 \leq \left(\frac{x_{e,h}}{D}\right)_{tur} \leq 60$$

We shall assume turbulent fully developed flow for $(x_{e,h}/D) > 10$.

Mean velocity: Because velocity varies over the cross section, it is necessary to work with a mean velocity u_m .

$$\dot{m} = \rho u_m A_c = \rho u_m \pi D^2/4, \quad Re_D = \frac{4\dot{m}}{\pi D \mu} \quad (3.1)$$

$$u_m = \frac{\int_{A_c} \rho u(r) dA_c}{\rho A_c} = \frac{2\pi \rho \int_0^{r_0} u(r) r dr}{\rho \pi r_0^2} = \frac{2}{r_0^2} \int_0^{r_0} u(r) r dr \quad (3.2)$$

Velocity profile in a fully developed pipe flow

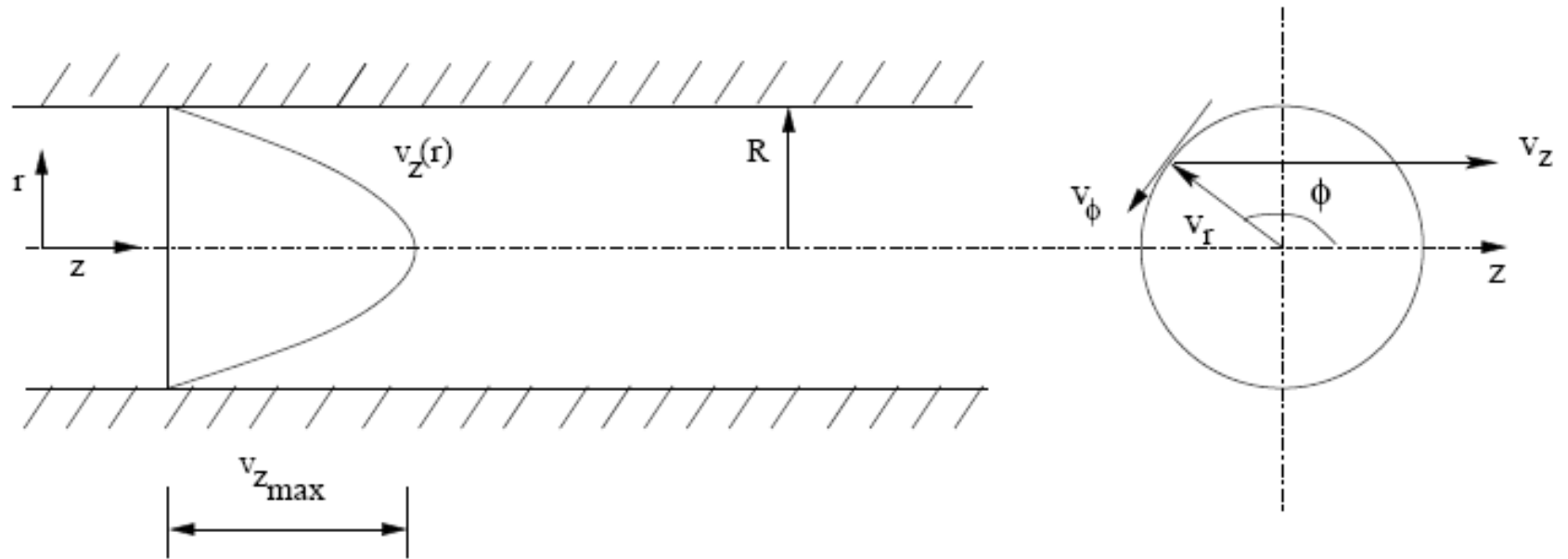


Figure 3.2: Hagen-Poiseuille flow through a pipe

The fully developed pipe flow is often called Hagen-Poiseuille flow (Figure 3.2). The general governing equations are:

Continuity

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z} = 0 \quad (3.3)$$

r momentum

$$\begin{aligned} & \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\phi}{r} \frac{\partial v_r}{\partial \phi} - \frac{v_\phi^2}{r} + v_z \frac{\partial v_r}{\partial z} \\ &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \phi^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_\phi}{\partial \phi} \right) \end{aligned} \quad (3.4)$$

ϕ momentum

$$\begin{aligned} & \frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\phi}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + v_z \frac{\partial v_\phi}{\partial z} \\ &= -\frac{1}{\rho} \frac{1}{r} \frac{\partial p}{\partial \phi} + \nu \left(\frac{\partial^2 v_\phi}{\partial r^2} + \frac{1}{r} \frac{\partial v_\phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{\partial^2 v_\phi}{\partial z^2} - \frac{v_\phi}{r^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \phi} \right) \end{aligned} \quad (3.5)$$

z momentum

$$\begin{aligned} & \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\phi}{r} \frac{\partial v_z}{\partial \phi} + v_z \frac{\partial v_z}{\partial z} \\ &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \phi^2} + \frac{\partial^2 v_z}{\partial z^2} \right) \end{aligned} \quad (3.6)$$

For fully developed flow, v_z is only non trivial component. $v_r = v_\phi = 0$. From continuity, $(\partial v_z / \partial z) = 0$ therefore, $v_z = v_z(r, \phi, t)$

Under steady state, $\partial (\text{anything}) / \partial t = 0$; flow is axisymmetric, $(\partial [\text{any variable}] / \partial \phi) = 0$

Therefore,

$$v_z = v_z(r)$$

From r and ϕ momentum equation, we get $p \neq p(r, \phi)$; $p = p(z)$ only

From z momentum equation, we get

$$\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} = \frac{1}{\mu} \frac{dp}{dz}$$

or,

$$\frac{1}{r} \frac{d}{dr} \left[r \frac{dv_z}{dr} \right] = \frac{1}{\mu} \frac{dp}{dz} \quad (3.7)$$

or,

$$r \frac{dv_z}{dr} = \frac{1}{\mu} \frac{dp}{dz} \frac{r^2}{2} + C_1$$

or,

$$\frac{dv_z}{dr} = \frac{1}{\mu} \frac{dp}{dz} \frac{r}{2} + \frac{C_1}{r}$$

or,

$$v_z = \frac{1}{4\mu} \frac{dp}{dz} r^2 + C_1 \ln r + C_2 \quad (3.8)$$

The boundary conditions are:

at $r = 0$, v_z is finite $\Rightarrow C_1 = 0$

at $r = r_0$, $v_z = 0 \Rightarrow 0 = \frac{1}{4\mu} \frac{dp}{dz} r_0^2 + C_2$

$$C_2 = -\frac{1}{4\mu} \frac{dp}{dz} r_0^2$$

$$v_z = -\frac{1}{4\mu} \left(\frac{dp}{dz} \right) r_0^2 \left[1 - \left(\frac{r}{r_0} \right)^2 \right]$$

Using, $v_z \leftrightarrow u$ and $z \leftrightarrow x$,

$$u(r) = -\frac{1}{4\mu} \left(\frac{dp}{dx} \right) r_0^2 \left[1 - \left(\frac{r}{r_0} \right)^2 \right] \quad (3.9)$$

$u = u_{max}$ at $r = 0$

$$u_{max} = -\frac{1}{4\mu} \left(\frac{dp}{dx} \right) r_0^2 \quad (3.10)$$

$$u_m = \frac{2}{r_0^2} \int_0^{r_0} -\frac{r_0^2}{4\mu} \left(\frac{dp}{dz} \right) \left[1 - \left(\frac{r}{r_0} \right)^2 \right] r dr$$

or,

$$u_m = \frac{2}{r_0^2} \left(\frac{r_0^2}{4\mu} \frac{dp}{dx} \right) \left[\frac{r^2}{2} - \frac{r^4}{4r_0^2} \right]_0^{r_0} = -\frac{1}{8\mu} \left(\frac{dp}{dx} \right) r_0^2 = \frac{u_{max}}{2} \quad (3.11)$$

or,

$$u_m = \frac{1}{2} u_{max} \quad \text{and} \quad \frac{u(r)}{u_m} = 2 \left[1 - \left(\frac{r}{r_0} \right)^2 \right] \quad (3.12)$$

Darcy friction factor

$$h_f = \frac{\Delta p}{\rho g} = \frac{f L u_m^2}{2 D g} \quad (3.13)$$

or,

$$f = \frac{(\Delta p / L) D}{\rho u_m^2 / 2} = \frac{(-dp/dx) D}{(\rho u_m^2 / 2)} \quad (3.14)$$

Fanning's friction factor

$$C_f = \frac{\tau_w}{(\rho u_m^2 / 2)} \quad (3.15)$$

$$C_f = \frac{(-dp/dx) (D/4)}{(\rho u_m^2 / 2)} \quad (3.16)$$

As such,

$$\tau_w = -\mu \frac{\partial u}{\partial r} \Big|_{r=r_0}$$

$$\tau_w = -\mu \left(+\frac{1}{4\mu} \frac{dp}{dx} \right) 2r_0^2 \frac{r_0}{r_0^2} = -\frac{1}{2} \frac{dp}{dx} r_0 = \frac{1}{2} \left(-\frac{dp}{dx} \right) \frac{D}{2}$$

Therefore, $C_f = f/4$ for fully developed laminar flow

$$f = \frac{(-dp/dx) (D)}{(\rho u_m^2/2)}$$

Substitute

$$u_m = -\frac{1}{8\mu} \frac{dp}{dx} r_0^2, \quad Re_D = \frac{\rho u_m D}{\mu}$$

or,

$$f = \frac{8\mu u_m D / r_0^2}{\rho u_m^2 / 2} = \frac{64}{\frac{\rho u_m D}{\mu}} = \frac{64}{Re_D} \quad (3.17)$$

fully developed laminar flow $f = \frac{64}{Re_D}$

For fully developed turbulent flow, the analysis is quite involved. At this stage, we may rely on the experimental results, Moody diagram (Figure 3.3). Friction factor is a function of Reynolds number and surface roughness. Correlations that reasonably approximate the smooth surface:

$$f = 0.316 Re^{-1/4} \dots\dots Re_D \leq 2 \times 10^4 \quad (3.18)$$

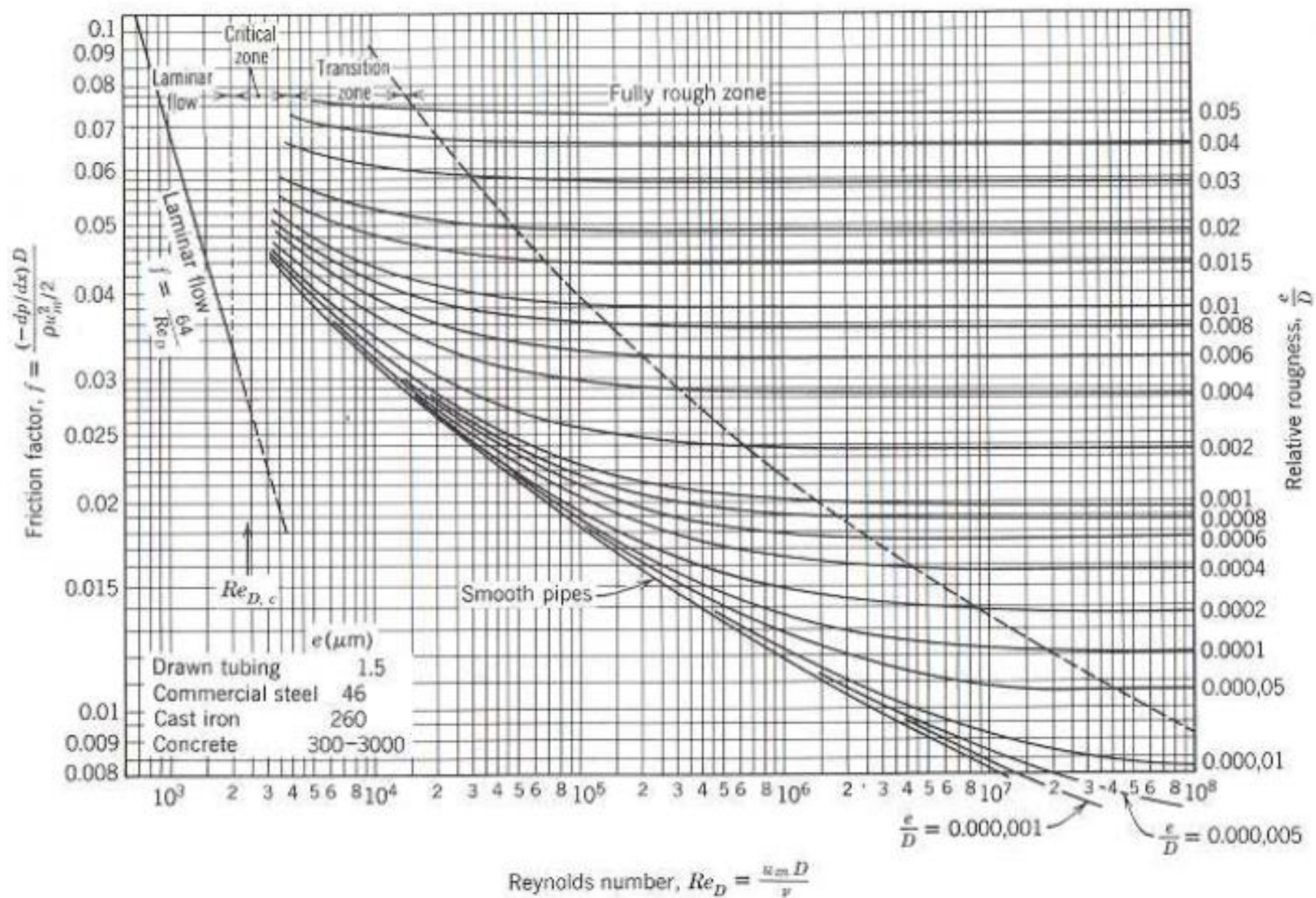


Figure 3.3: Friction factors for pipes

$$f = 0.184 Re^{-1/5} \text{.....} Re_D \geq 2 \times 10^4 \quad (3.19)$$

Note that f , hence (dp/dx) is constant in fully developed region

$$f = \frac{-(dp/dx)D}{\rho u_m^2/2}$$

$$\Delta p = - \int_{p_1}^{p_2} dp = \int_1^2 f \frac{\rho u_m^2}{2D} dx = \frac{f \rho u_m^2}{2D} (x_2 - x_1) = \frac{f \rho u_m^2}{2D} L$$

$f = \frac{64}{Re_D}$ in laminar flow, $f = 0.316 Re_D^{-1/4}$ for $Re_D \leq 2 \times 10^4$ (turbulent flow) and

$f = 0.184 Re_D^{-1/5}$ for $Re_D > 2 \times 10^4$ (for turbulent flow)

However, from the above relation we get back $\Delta p / \rho g = \frac{f L u_m^2}{2 D g} = h_f$

Thermal Considerations during Internal Flows

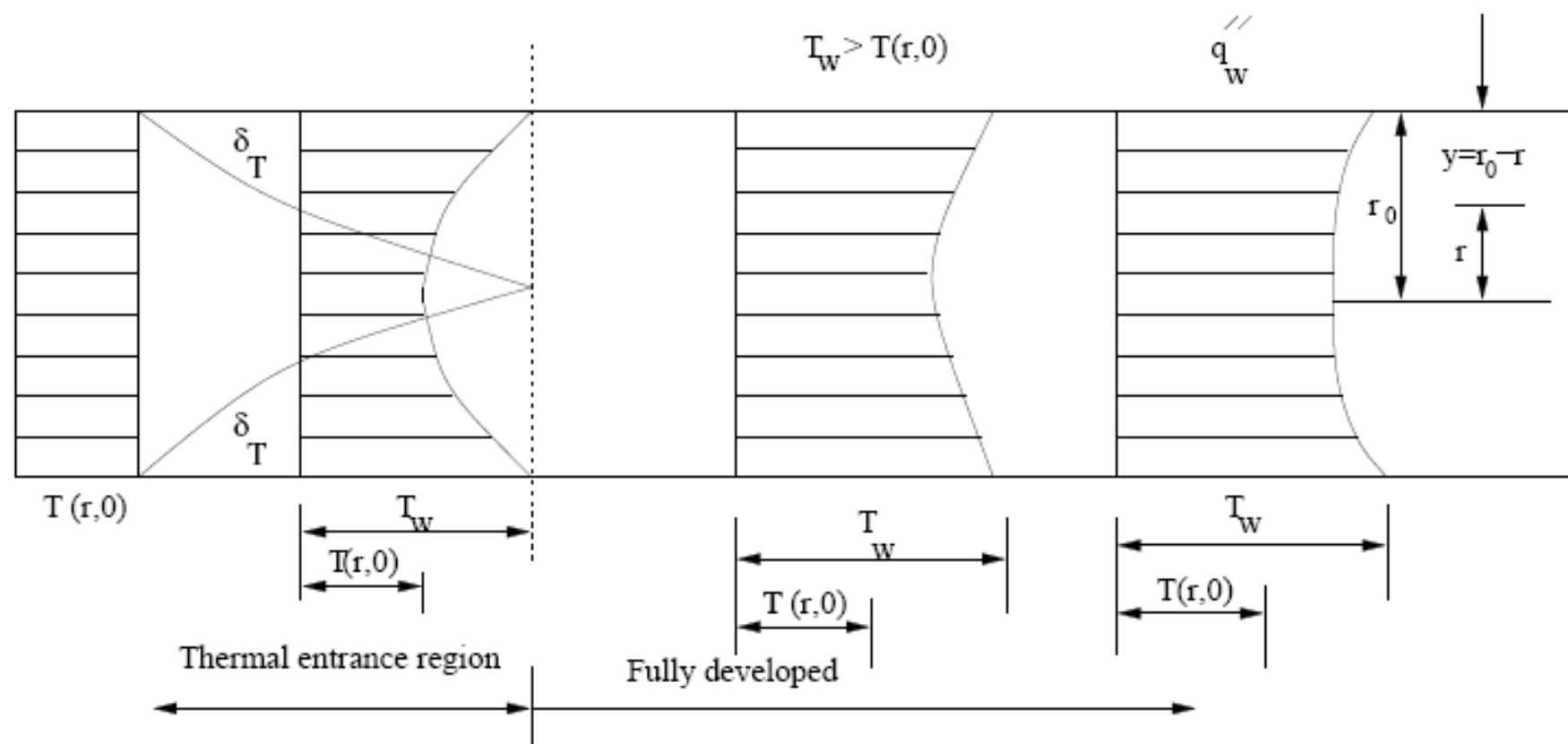


Figure 3.4: Thermally developing duct flow

Tube surface condition: either a uniform wall temperature (UWT) condition or a uniform wall heat flux condition (UWH).

A fully developed temperature profile differs according to the surface conditions. For both surface conditions, however, the amount by which the fluid temperature exceeds the entrance temperature increases with increasing x (Figure 3.4). In order to study the development of thermal boundary layer, a hydrodynamically fully developed flow may be considered (Figure 3.5).

1. For laminar flow, the thermal entrance length may be expressed as

$$\left(\frac{x_{e,t}}{D} \right)_{lam} \approx \begin{cases} 0.033 Re_D Pr & \text{for UWH} \\ 0.043 Re_D Pr & \text{for UWT} \end{cases} \quad (3.20)$$

we can say, $(x_{e,t}/D) = 0.05 Re_D Pr$

Comparing this with hydrodynamics boundary layer, it can be said that if $Pr > 1$, hydrodynamic boundary layer grows more rapidly.

$Pr > 1$, $x_{e,h} < x_{e,t} \Rightarrow \delta > \delta_T$ at any section; even otherwise $Pr \sim (\delta/\delta_T)^{1/2}$.

Therefore, for $Pr > 1$, δ has to be $> \delta_T$.

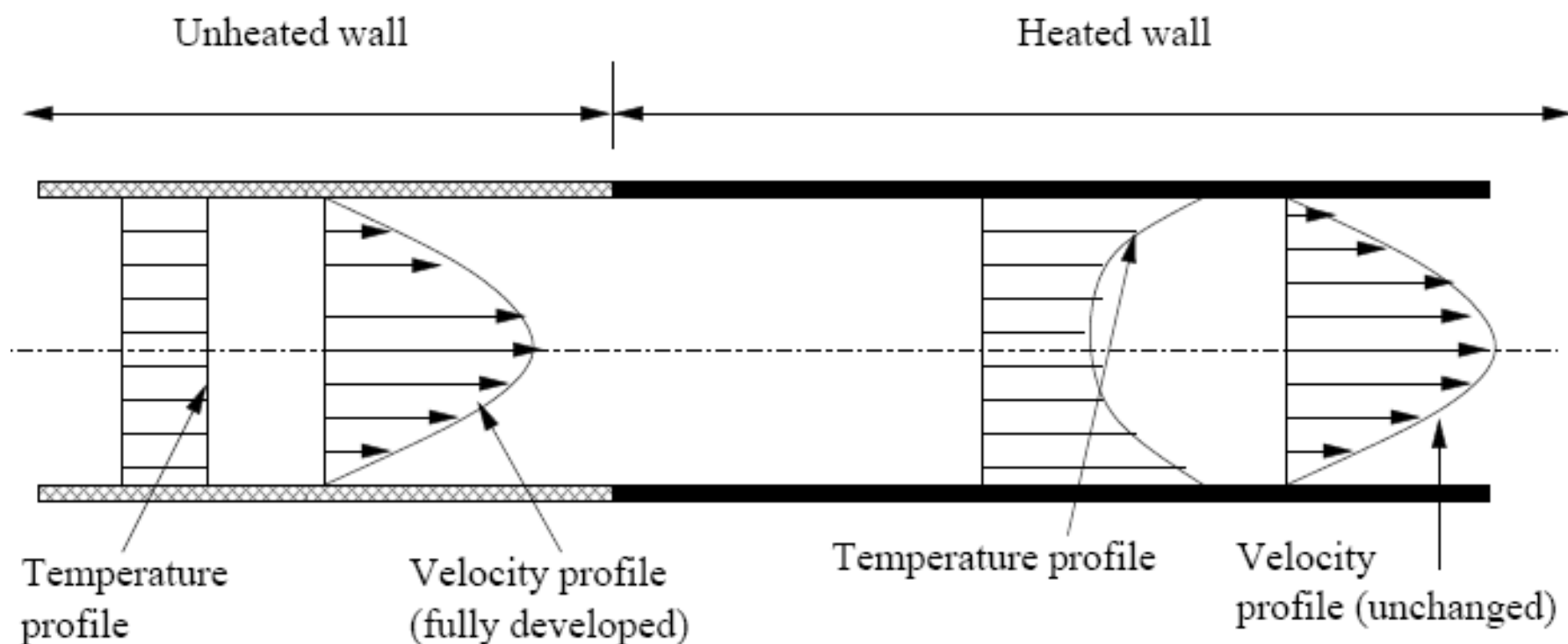


Figure 3.5: Developing velocity and temperature fields

2. Inverse is true for $Pr < 1, Pr \sim (\delta/\delta_T)^{1/3}$ and consequently δ_T is greater than δ .

However, for extremely High Prandtl number fluids (such as oil), $Pr \geq 100$, $x_{e,h} \ll x_{e,t}$. Throughout the thermal entrance region, hydrodynamically fully developed velocity profile can be assumed

For turbulent flow, conditions are nearly independent of Prandtl number. We can accept $\left(\frac{x_{e,t}}{D}\right)_{tur} \approx 10$. Bulk mean temperature at any section (x) is given by

$$T_m = \frac{2}{u_m r_0^2} \int_0^{r_0} u(r) T(r) r dr \quad (3.21)$$

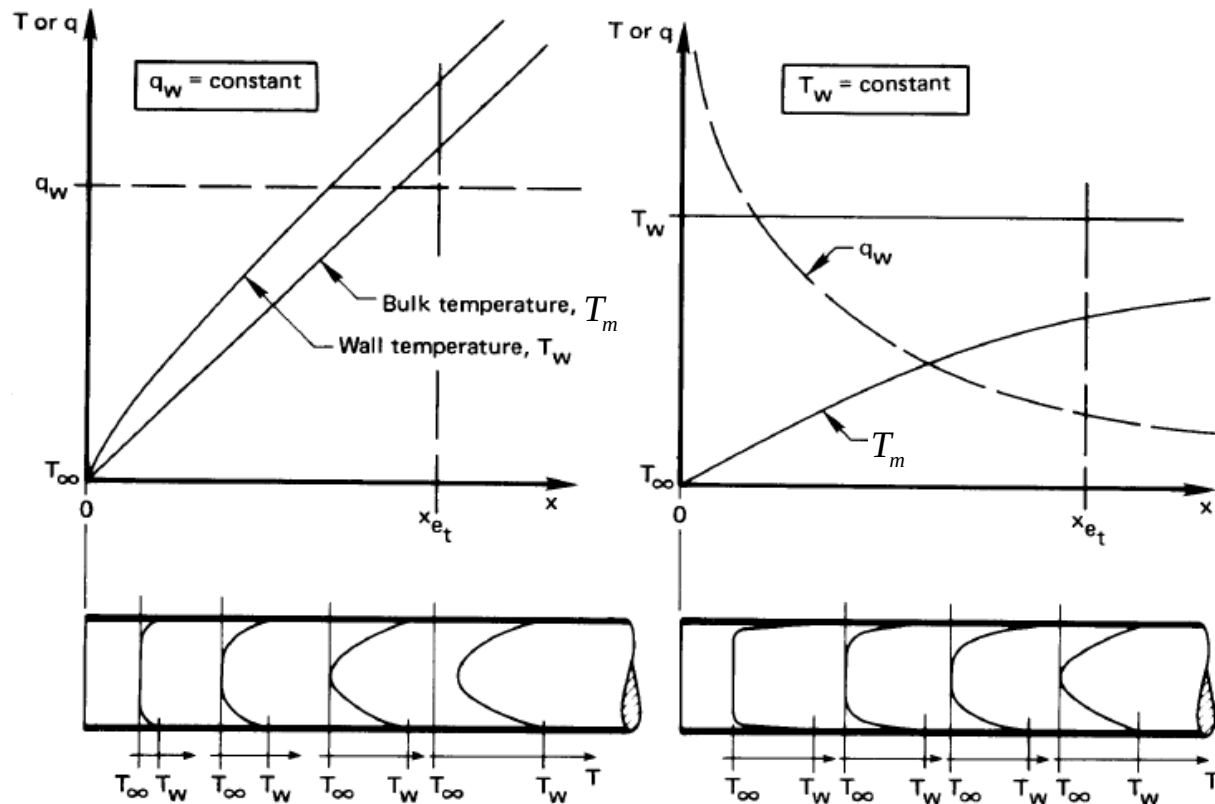
3.3.1 Newton's Law of Cooling for Internal Flows

$$q_w'' = h(T_w - T_m) \quad (3.22)$$

The bulk mean temperature T_m is a convenient reference temperature for internal flows. T_m behaves like T_∞ for external flows. Essential difference is T_m varies in the flow direction. T_m increases in the flow direction if heat transfer is from surface to fluid. It is very important to know the variation of T_m under various hydrodynamic and thermal conditions, since the energy transfer is dependent on it.

Fully Developed Thermal Conditions

Heat transfer between the surface and the fluid results in a condition that the fluid temperature must continue to change with x . One might wonder about existence fully developed thermal conditions under such a condition. The situation is different from the hydrodynamic case, for which $(\partial u / \partial x) = 0$ for the fully developed flow. In the presence of heat transfer, (dT_m / dx) as well as $(\partial T / \partial x)$ at any radial location are not zero.





Thank you

